DWARAKA DOSS GOVERDHAN DOSS VAISHNAV COLLEGE (AUTONOMOUS) Arumbakkam, Chennai - 600 106.



POST GRADUATE & RESEARCH DEPARTMENT OF MATHEMATICS (M Sc Mathematics) Programme Code: 21

Academic Year 2018-19 SYLLABUS (Choice Based Credit System)

PRINCIPAL Dwaraka Doss Goverdhan Doss Vaishnav College Arumbakkam, Chennai - 600106. R.VENKATARAMANAN M.Sc., M.Phil., ASSOCIATE PROFESSOR AND HEAD P.G. AND RESEARCH DEPT OF MATHEMATICS DWARAKADOSS GOVERDHANDOSS VAISHNAV COLLEGE CHENNAI - 600 106

| Title of the Course | | Algebra I | |
|---------------------|--------------|--------------|-------|
| Class | M Sc Maths | Subject Code | 21101 |
| Year | Ι | Semester | Ι |
| Course | Core Paper I | Credit | 4 |

UNIT-I Counting Principles, Class Equations for finite groups and its applications, Sylow's Theorems (For theorem 2.12.1, First proof only)

UNIT-II Solvable Groups, Direct Products, Finite Abelian Groups, Modules

UNIT-III: Linear Transformations, Canonical form, Triangular form, Nilpotent Transformation

UNIT-IV : Jordan form, rational canonical form

UNIT-V : Trace and Transpose, Hermitian, Unitary, Normal Transformations, real quadratic forms

Recommended Text Book:

I.N. Herstein, Topics in Algebra, Second Edition, Wiley (2002).

| Unit 1 | Sections 2.11 and 2.12 (Omit Lemma 2.12.5) |
|--------|---|
| II:4 3 | Sections 5.7 (Lemma 5.7.1, Lemma 5.7.2, Theorem 5.7.1), Sections 2.13 and 2.14, |
| Unit 2 | (Theorem 2.14.1 only), Section 4.5 |
| Unit 3 | Sections 6.4, 6.5 |
| Unit 4 | Sections 6.6, 6.7 |
| Unit 5 | Sections 6.8, 6.10 and 6.11 (Omit Section 6.9) |

Reference Books:

1.M. Artin, Algebra, Prentice Hall of India, 1991.

- 2.P.B. Bhattacharya, S.K. Jain and S.R. Nagupaul, Basic Abstract Algebra, (Second Edition), Cambridge University Press, Indian Edition, 1997.
- 3.*I.S. Luther* and *I.B.S. Passi*, Algebra Volume I Groups (1996), Volume II Rings (1999), Narosa Publishing House, New Delhi

4.D.S. Dummit and R.S. Foote, Abstract Algebra. Second Edition, Wiley, 2002.

5.N. Jacobson, Basic Algebra, Vol I and II, Hindustan Publishing Company, New Delhi.

Course Outcomes: At the end of the Course, the Student will be able to

> Explain Quadratic forms with suitable examples and apply them to problems

| Title of the Course | | Real Analysis I | |
|---------------------|---------------|-----------------|-------|
| Class | M Sc Maths | Subject Code | 21102 |
| Year | Ι | Semester | Ι |
| Course | Core Paper II | Credit | 4 |

UNIT-I : Elements of Point set Topology Definition of Adherent points, Accumulation points, Closed sets and adherent points with suitable illustrations, Constructing the proofs of: The Bolzano-Weierstrass theorem ,The Cantor intersection theorem ,The Lindeloff covering theorem , The Heine Borel covering theorem, Formulating the concept of Compactness in \mathbb{R}^n with suitable examples.

UNIT-II : Limits and Continuity Definition and Explanation of Connectedness, Components of a metric space, arcwise connectedness, uniform continuity, Fomulating the concept of compact sets through uniform continuity, Construction of the proof of fixed point theorem with respect to contraction mappings.

UNIT-III : Functions of bounded variation Classifying and explaining the Properties of monotonic functions, Explanation of Functions of bounded variation, Total variation with suitable illustrations. Constructing the proofs of Additive property of total variation, Total variation on [a, x] as a function of x, Functions of bounded variation expressed as the difference of two increasing functions, Continuous functions of bounded variation.

Defining the Infinite Series and Infinite products, Explaining Multiplication of series and Illustrating the concept of Cesaro summability with examples and proofs.

UNIT-IV: The Riemann - Stieltjes Integral Definition of the Riemann - Stieltjes integral, Constructing the proofs of Linear Properties, Integration by parts, Change of variable in a Riemann-Stieltjes integral, Reduction to a Riemann Integral, Step functions as integrators, Reduction of a Riemann – Stieltjes integral to a finite sum, Euler's summation formula. Definition of Monotonically increasing integrators upper and lower integrals and classifying Riemann's condition with equivalent conditions.

Recommended Text Book:

Tom M.Apostol, Mathematical Analysis, 2nd Edition, Narosa, 1989.

| Unit 1 | Sections 3.6 to 3.12 |
|--------|--|
| Unit 2 | Sections 4.16 to 4.21 |
| Unit 3 | Sections 6.1 to 6.8 and 8.24, 8.25 |
| Unit 4 | Sections 7.1 to 7.10 |
| Unit 5 | Section 7.11, 7.13 to 7.20 (Omit Section 7.12) |

REFERENCE BOOKS:

1. Bartle, R.G. Real Analysis, John Wiley and Sons Inc., 1976.

- 2. *Rudin, W.* Principles of Mathematical Analysis, 3rd Edition. McGraw Hill Company, New York, 1976.
- 3. Malik, S.C. and Savita Arora, Mathematical Anslysis, Wiley Eastern Limited. New Delhi, 1991.

- Explain point set topology and functions of bounded variation with suitable examples and apply them to problems
- > Apply the concept of Riemann Stieltjes integrals.

| Title of the Course | | Probability Theory | |
|---------------------|----------------|--------------------|-------|
| Class | M Sc Maths | Subject Code | 21103 |
| Year | Ι | Semester | Ι |
| Course | Core Paper III | Credit | 4 |

UNIT-I: Random Events and Random Variables: Random events – Probability axioms – Combinatorial formulae – conditional probability – Bayes Theorem – Independent events – Random Variables – Distribution Function – Joint Distribution – Marginal Distribution – Conditional Distribution – Independent random variables – Functions of random variables.

UNIT-II: Parameters of the Distribution : Expectation- Moments – The Chebyshev Inequality – Absolute moments – Order parameters – Moments of random vectors – Regression of the first and second types.

Chapter 3 : Sections 3.1 to 3.8 (Generalization of Regression line of second type is omitted) UNIT-III: Characteristic functions : Properties of characteristic functions – Characteristic functions and moments – semi invariants – characteristic function of the sum of the independent random variables – Determination of distribution function by the Characteristic function – Characteristic function of multidimensional random vectors – Probability generating functions. **UNIT-IV : Some Probability distributions:**

Discrete distributions: One point, two point, Binomial – Polya – Hypergeometric – Poisson distributions

Continuous distributions – Uniform – Normal – Gamma – Beta – Cauchy and Laplace distributions.

UNIT-V:Limit Theorems : Stochastic convergence – Bernaulli law of large numbers – Levy-Cramer Theorems (only one part of the theorem can be asked) – De Moivre-Laplace Theorem – Poisson, Chebyshev, Khintchine Weak law of large numbers – Lindberg Theorem – LapunovTheroem – Borel-Cantelli Lemma – Kolmogorov Inequality and Kolmogorov Strong Law of large numbers.

Recommended Text Book:

M. Fisz, Probability Theory and Mathematical Statistics, John Wiley and Sons, New York, 1963.

| Chapter 1 | Sections 1.1 to 1.7 |
|-----------|---|
| Chapter 2 | Sections 2.1 to 2.9 |
| Chapter 3 | Sections 3.1 to 3.8 (Generalization of Regression line of second type is omitted) |
| Chapter 4 | Sections 4.1 to 4.7 |
| Chapter 5 | Section 5.1 to 5.10 (Omit Section 5.11), (Omit examples 5.5.1 and 5.5.2) |
| Chanton (| Sections 6.1 to 6.4, 6.6 to 6.9, 6.11 and 6.12. (Omit Sections 6.5, 6.10,6.13 to 6.15), |
| Chapter o | (omit example 6.9.1) |

Reference Book:

V.K.Rohatgi, An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern New Delhi, 1988(3rd Edition)

- Restate the probability concept with reference to Borel fields and categorize the discrete and continuous distributions by analyzing their characterization and structure.
- > Demonstrate the application of Limit theorem in various situations.
- > Prepare a probability model for any given real life situation through survey.

| Title of the Course | | Graph Theory | |
|---------------------|---------------|--------------|-------|
| Class | M Sc Maths | Subject Code | 21104 |
| Year | Ι | Semester | Ι |
| Course | Core Paper IV | Credit | 4 |

UNIT-I : Graphs, Subgraphs and Trees : Graphs and simple graphs – Graph Isomorphism – The Incidence and Adjacency Matrices – Subgraphs – Vertex Degrees – Paths and Connection – Cycles – Trees – Cut Edges and Bonds – Cut Vertices.

UNIT-II : Connectivity, Euler tours and Hamilton Cycles : Connectivity – Blocks – Euler tours – Hamilton Cycles.

UNIT-III:Matchings, Edge Colourings : Matchings – Matchings and Coverings in Bipartite Graphs – Edge Chromatic Number – Vizing's Theorem.

UNIT-IV :Independent sets and Cliques, Vertex Colourings: Independent sets – Ramsey's Theorem – Chromatic Number – Brooks' Theorem – Chromatic Polynomials.

UNIT-V:Planar graphs : Plane and planar Graphs – Dual graphs – Euler's Formula – The Five-Colour Theorem and the Four-Colour Conjecture.

Recommended Text:

J.A.Bondy and U.S.R. Murthy, Graph Theory and Applications, Macmillan, London, 1976.

| UNIT I | Chapter 1 Sections 1.1 – 1.7, Chapter 2 Sections 2.1 – 2.3 |
|----------|--|
| UNIT II | Chapter 3 Sections 3.1 – 3.2, Chapter 4 Sections 4.1 – 4.2 |
| UNIT III | Chapter 5 Sections 5.1 – 5.2, 6apter 4 Sections 6.1 – 6.2 |
| UNIT IV | Chapter 7 Sections 7.1 – 7.2, Chapter 8, Sections 8.1 – 8.2, 8.4 |
| UNIT V | Chapter 9, Sections 9.1 – 9.3, 9.6 |

- > Apply the different concepts of Graph Theory and its application in day today life.
- Enable the students to model the real-world problems into Graph Theory problems and get solutions.

| Title of the Course | | Numerical Analysis | |
|---------------------|-----------------------|--------------------|-------|
| Class | M Sc Maths | Subject Code | 21105 |
| Year | Ι | Semester | Ι |
| Course | Elective I - Paper IV | Credit | 5 |

UNIT-I : Non Linear equation Bisection method, The Secant method, **Regulafalsi method**, **Newton Raphson method**, The fixed point method, Muller's Method. Newton's method for multiple roots. System of non linear equations by Newton's method and fixed point method.

UNIT-II : Interpolation: Lagrange's formula, Newton's divided difference formula, Newton's forward and backward Interpolation formula.

Numerical Differentiation: Derivatives based on Newton's forward and backward Interpolation formula.

UNIT-III: Numerical Integration: Basic Trapezoidal rule, Composite Trapezoidal rule, Basic Simpson's one third rule, Composite Simpson's one third rule, Basic Simpson's three eighth rule. Numerical double integration with constant limits by composite trapezoidal and composite Simpson's one third rule.

UNIT-IV : Numerical solution of ordinary differential equations: Difference equation, Taylor's series method, Euler's method, Runge-kutta method (fourth order only). Predictor-corrector methods -Milne's method and Adam's method.

UNIT-V: Numerical solution of partial differential equation: Introduction, Solution of Laplace's equation $U_{xx} + U_{yy} = 0$ and Poisson's equation by Jacobi's and Gauss Seidel method.

Solution of parabolic heat conduction equation $U_{xx} = CU_t$ by Bender Schmidtrecurrence relation and Crank Nickolson formula.

Recommended Text Book:

- 1) *Devi Prasad*, An Introduction to Numerical Analysis, (Third edition) Narosa Publishing house, New Delhi,2008.
- 2) *S.S.Sastry*, Introductory methods of Numerical Analysis (Fourth Edition), PHI Learning Pvt Ltd, NewDelhi, 2009

| UNIT | AUTHOR | CHAPTER- SECTIONS |
|----------|-------------|---|
| Unit I | Devi Prasad | Chapter 2 |
| Unit II | Devi Prasad | Chapter 4- 4.1 to 4.3 ; Chapter 5 - 5.1 |
| Unit III | Devi Prasad | Chapter 5 - 5.3, 5.7.1, 5.7.3 |
| Unit IV | Devi Prasad | Chapter 6- 6.1 to 6.4 |
| Unit V | S.S. Sastry | Chapter 8- 8.1 ,8.2,8.3.1.8.3.2,8.4 |

Reference Books:

1) *K.Sankara Rao*, Numerical Methods for Scientists and Engineers, (Third edition) Prentice Hall of India Pvt Ltd New Delhi 2007.

2) Jain, Iyengar, Numerical Methods, (Fifth edition) New age Publishers, 2010.

- Solve nonlinear equations and system of nonlinear equations.
- Apply Interpolation for unequal and equal intervals and Compute derivatives using numerical differentiation.
- > Develop numerical techniques to solve calculus problems.

| Title of the Course | | Algebra II | Algebra II | |
|---------------------|---------------|--------------|------------|--|
| Class | M Sc Maths | Subject Code | 21207 | |
| Year | Ι | Semester | II | |
| Course | Core Paper VI | Credit | 4 | |

UNIT-I : Extension Fields

Extension Fields, Transcendence of e

UNIT-II : Roots of Polynomials

Roots of Polynomials, roots of polynomials with integer and rational coefficients, more about roots.

UNIT-III : Galois Theory

Galois Theory and proving theorems related to Galois Theory.

UNIT-IV: Finite Fields and Division Rings

Finite Fields, Examples, Theorems, Wedderburn's Theorem for finite division rings.

UNIT-V : Solvability by Radicals and Four Squares Theorem

Solvability by Radicals, Frobenius Theorem, Integral Quaternions, Four Squares Theorem

Recommended Text Book:

I.N. Herstein, Topics in Algebra, Second Edition, Wiley (2002).

| Unit 1 | Sections 5.1 to 5.2 |
|--------|---|
| Unit 2 | Sections 5.3 and 5.5 (Omit Section 5.4) |
| Unit 3 | Section 5.6 |
| Unit 4 | Sections 7.1, 7.2 |
| Unit 5 | Sections 7.3, 7.4 |

Reference Books:

1.M. Artin, Algebra, Prentice Hall of India, 1991

- 2.P.B. Bhattacharya, S.K. Jain and S.R. Nagupaul, Basic Abstract Algebra, (Second Edition), Cambridge University Press, Indian Edition, 1997
- 3.D.S. Dummit and R.S. Foote, Abstract Algebra, Second Edition, Wiley, 2002
- 4.N. Jacobson, Basic Algebra, Vol I and II, Hindustan Publishing Company, New Delhi

- Restate the ideas of Extension Fields, Galois Theory, Finite Fields, Wedderburn's Theorem for finite division rings and Four Squares Theorem.
- Understand Algebra and Linear Algebra at Advanced level and apply it in various branches of Engineering and Science.

| Title of the Course | | Real Analysis II | |
|---------------------|----------------|------------------|-------|
| Class | M Sc Maths | Subject Code | 21208 |
| Year | Ι | Semester | II |
| Course | Core Paper VII | Credit | 4 |

UNI

T-I : Measure on the Real line: Definition of Lebesgue Outer Measure, Measurable sets with suitable examples. Developing Regularity conditions and constructing the proofs regarding measurable sets.

UNIT-II : Measure on the real line and Integration of Functions of a Real variable : Definition of Measurable Functions, Borel and Lebesgue Measurability, Explanation of Integration of Non- negative functions and constructing proofs of properties of such functions.

Unit -III: Integration of Functions of a Real variable: Definition of the general integral, Developing Integration of series, Classifying and Distinguishing between the concepts of Riemann and Lebesgue integrals.

UNIT-IV : Fourier Series and Fourier Integrals: Restating Orthogonal system of functions, Constructing the proofs of: The theorem on best approximation, The Fourier series of a function relative to an orthonormal system, Properties of Fourier Coefficients, The Riesz-Fischer Theorem, Developing the convergence and representation problems in for trigonometric series, Explanation of the proofs of: The Riemann - Lebesgue Lemma, The Dirichlet Integrals, An integral representation for the partial sums of Fourier series, Riemann's localization theorem, Sufficient conditions for convergence of a Fourier series at a particular point, Explanation of Cesaro summability of Fourier series, Consequences of Fejes's theorem, Constructing the proof of Weierstrass approximation theorem

UNIT-V : Multivariable Differential Calculus: Definition of the concepts like Directional derivative, Continuity, The total derivative, The total derivative expressed in terms of partial derivatives with illustration of suitable examples, Definition of the matrix of linear function, The Jacobian matrix. Construction of the proofs of: The chain rule, The mean - value theorem for differentiable functions, Sufficient condition for differentiability, Sufficient condition for equality of mixed partial derivatives, Taylor's theorem for functions of \mathbb{R}^n to \mathbb{R}^1

Recommended Text Books:

1. *G.de Barra*, Measure Theory and Integration, New Age International, 2003(for Units 1, 2 and 3)

| Unit 1 | Sections 2.1 to 2.3 (Chapter 2 of de Barra Book) |
|--------|---|
| Unit 2 | Sections 2.4, 2.5, 3.1 (Chapters 2 and 3 of de Barra Book) |
| Unit 3 | Sections 3.2 to 3.4 (Chapter 3 of de Barra Book) |
| Unit 4 | Sections 11.1 to11.5 (Chapter 11 of Apostol Book) |
| TI | Section 12.1 to 12.5 and 12.8, 12.9, 12.11 to 12.14 (Omit Sections 12.6, 12.7, 12.10, |
| Unit 5 | 12.13) (Chapter 12 of Apostol Book) |

2. *Tom M.Apostol*, Mathematical Analysis, 2nd Edition, Narosa 1989 (for Units 4 and 5)

Reference Books:

1. Burkill, J.C., The Lebesgue Integral, Cambridge University Press, 1951.

3. Royden, H.L. Real Analysis, Macmillan Pub. Company, New York, 1988.

4. Rudin W., Principles of Mathematical Analysis, McGraw Hill Company, New York, 1979.

5. Malik, S.C. and Savita Arora, Mathematical Analysis, Wiley Eastern Limited, New Delhi, 1991.

Course Outcomes: At the end of the Course, the Student will be able to

Analyze the ideas of Measure Theory, Lebesgue Integrals, Functions of Several Variables and Fourier Series, Fourier Integrals and their convergence aspects.

^{2.} Munroe, M.E., Measure and Integration, Addison-Wesley, Mass. 1971.

> Apply The Riesz-Fischer Theorem and Taylor's theorem for functions of R^n to R^1 .

| Title of the Course | | Differential Equations | |
|---------------------|-----------------|------------------------|-------|
| Class | M Sc Maths | Subject Code | 21209 |
| Year | Ι | Semester | II |
| Course | Core Paper VIII | Credit | 4 |

UNIT-I : Linear differential equations of higher order Introduction-Linear dependence and Wronskian-linear equations-Method of variation of parameters-Two useful formulae-Homogeneous linear equation with constant coefficients

UNIT-II : Solutions in Power series Introduction-second order linear equations with ordinary points- Legendre equation and Legendre Polynomials-Second order equations with regular singular points-Bessels equation .

UNIT-III:Systems of Linear Differential Equation Introduction-Systems of first order equations-Existence and uniqueness theorem-Fundamental matrix-Non homogeneous linear systems-Linear systems with constant coefficients.

UNIT-IV : Second order Partial differential equation. Methods for solving Linear PDE, Classification and canonical form

Parabolic differential Equations: Solving One dimensional heat conduction equation $\frac{\partial u}{\partial x} = k \frac{\partial^2 u}{\partial x^2}$

by Separation of variables.

Hyperbolic differential equation – one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separation of

variables

UNIT-V:Elliptic Differential Equations: Solving Two dimensional Laplace's equation $\nabla^2 u = 0$

by Separation of Variables , Laplace equation in cylindrical and Spherical coordinates, Interior and Exterior Dirichlets's problems for a circle

Recommended Text :

- 1. *S.G.Deo and V.Raghavendra*, Ordinary Differential Equations and Stability theory, Tata Mcgraw-Hill Publishing company Ltd.
- 2. J.N.Sharma and Keharsingh, Partial Differential Equations for Engineers and scientists, Narosa Publishing House, New Delhi, 2000

| Units | Text – Author Chapter | |
|----------|--|--|
| Unit I | IDeo & RaghavendraChapter 2 fully | |
| Unit II | II Deo & Raghavendra Chapter 3 fully | |
| Unit III | t III Deo & Raghavendra Chapter 4 Sections 4.1 to 4.6 | |
| Unit IV | IVJ. N. Sharma &Kehar SinghChapter 2, Sections2.1-2.3,2.4 ,2.4.1, Chapter 4, Section Chapter 5, Section 5.5 | |
| Unit V | J. N. Sharma & Kehar Singh Chapter 3, Sections 3.3 to 3.7 | |

- > Apply the fundamental concepts and the techniques for solving ordinary differential equations in standard forms and using power series.
- > Analyze non-homogeneous linear systems with constant coefficients.
- > To know the role of PDE in modern Mathematics.

> Solve standard second order PDE by variable separable method.

| Title of the Course | | Mathematical Statistics | |
|---------------------|---------------|-------------------------|-------|
| Class | M Sc Maths | Subject Code | 21210 |
| Year | Ι | Semester | II |
| Course | Core Paper IX | Credit | 4 |

UNIT-I : Sample Moments and their Functions: Notion of a sample and a statistic – Distribution functions of \overline{X} , and (\overline{X}, S) – Chi square distribution– Student t-distribution – Fisher's Z-distribution – Snedecor's F distribution– Distribution of sample mean from non-normal populations

UNIT-II : Significance Test : Concept of a statistical test – Parametric Tests for small samples and large samples – Chi square test – Independence Tests by contingency tables.

UNIT-III : Estimation : Preliminary notion – Consistency estimation – Unbiased estimates – Sufficiency – Efficiency – Asymptotically most Efficient estimates – methods of finding estimates – confidence Interval.

UNIT-IV : Analysis of Variance : One way classification and two-way Classification (theory only).

Hypotheses Testing: Poser functions – OC function- Most Powerful test – Uniformly most powerful test – unbiased test.

UNIT-V : Sequential Analysis : SPRT – Auxiliary Theorem – Wald's fundamental identity – OC function of SPRT – E(n) and Determination of A and B – Testing a hypothesis concerning p on 0-1 distribution and m in Normal distribution.

Recommended Text Book:

M. Fisz, Probability Theory and Mathematical Statistics, John Wiley and Sons, New York, 1963.

| Chapter 9 | Sections 9.1 to 9.8 |
|------------|---|
| Chapter 12 | 12.1 to 12.4 and 12.7 (omit 12.5, 12.6). |
| Chapton 12 | Sections 13.1 to 13.8 (Omit Section 13.9), (omit examples |
| Chapter 13 | 13.6.3, 13.6.4 and 13.7.2 to 13.7.4) |
| Chapter 15 | Sections 15.1 and 15.2 |
| Chapter 16 | Sections 16.1 to 16.5 (Omit Section 16.6 and 16.7),(omitexample 16.2.3) |
| Chapter 17 | Sections 17.1 to 17.9 (Omit Section 17.10) |

Reference Books:

V.K.Rohatgi, An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern New Delhi, 1988(3rd Edition)

- Summarize the importance of sampling distributions and their classifications.
- > Appraise the parameter value of any given distribution using statistical tools.
- Use the hypotheses testing technique to sample (Variable or Constant in size) collected from real life situation and compare the outcome with the existing one.

| Title of the Course | | Fuzzy Sets and Their Applications | |
|---------------------|----------------------|-----------------------------------|-------|
| Class | M Sc Maths | Subject Code | 21211 |
| Year | Ι | Semester | II |
| Course | Elective II- Paper X | Credit | 5 |

UNIT I:Fundamental Notions: Introduction, review of the notion of membership, the concept of a fuzzy subsets, dominance relations, simple operations on fuzzy subsets, sets of fuzzy subsets for **E** and **M** finite, properties of the set of fuzzy subsets, product and algebraic sum of two fuzzy subsets.

UNIT-II : Fuzzy Graphs: Introduction, fuzzy graphs, fuzzy relation composition of two fuzzy relation, fuzzy subsets induced by a mapping, conditioned fuzzy subsets, properties of fuzzy binary relation, transitive closure of fuzzy binary relation and paths in a finite fuzzy graph.

UNIT-III:Fuzzy Relations:Fuzzy pre order relation, similitude relation, similitude subrelations in a fuzzy preorder, antisymmetry, fuzzy order relations, antisymmetric relations without loops, dissimilitude relations, resemblance relation, various properties of similitude and resemblance, various properties of fuzzy perfect order relations and ordinary membership functions.

UNIT-IV : Fuzzy Logic: Introduction, characteristic function of a fuzzy subset, Polynomial forms, analysis of a function of fuzzy variables, logical structure of a function of fuzzy variables, composition of intervals, fuzzy propositions and their functional representations and the theory of fuzzy subsets and the theory of probability.

UNIT-V:The Laws of Fuzzy Composition: Introduction, review of the notion of a law of composition, law of Fuzzy internal composition, fuzzy groupoids, principal properties of fuzzy groupoids, fuzzy monoids, fuzzy external composition and operations of fuzzy numbers.

Recommended Text :

A.Kaufmann, Introduction to the theory of Fuzzy subsets, Vol.I, Academic Press, New York, 1975.

| UNIT | CHAPTER- SECTIONS |
|----------|---|
| Unit I | Chapter I: Sec. 1 to 8. |
| Unit II | Chapter II: Sec. 10 to 18 |
| Unit III | Chapter II: Sec. 19 to 29. |
| Unit IV | Chapter III:Sec.31 to 40 (Omit Sec. 37, 38 and 41). |
| Unit V | Chapter IV: Sec.43 to 49. |

Reference Books:

- 1. H.J.Zimmermann, Fuzzy Set Theory and its Applications, Allied Publishers, Chennai, 1996.
- 2. *George J.Klir and Bo Yuan*, Fuzzy sets and Fuzzy Logic- Theory and Applications, Prentice Hall India, New Delhi,2001.

- > Apply the concepts of fuzzy sets and fuzzy relations.
- > Apply analysis of function of fuzzy variable using fuzzy logic.

| Title of the Course | | Complex Analysis I | |
|---------------------|---------------|--------------------|-------|
| Class | M Sc Maths | Subject Code | 21313 |
| Year | II | Semester | III |
| Course | Core Paper XI | Credit | 4 |

UNIT I: Line integrals – Rectifiable arcs – Line integrals as functions of arcs – Cauchy's theorems for a rectangle – Cauchy theorem in a disk -- The Index of a point with respect to a closed curve – The Integral formula

Chapter 4 : Section 1 : 1.1 to 1.5

Chapter 4 : Section 2 : 2.1 to 2.2

UNIT-II: Higher derivatives -- Removable Singularities - Taylors's Theorem – Zeros and poles – The local Mapping – The Maximum Principle.

Chapter 4 : Section 2 : 2.3

Chapter 4 : Section 3 : 3.1 to 3.4

UNIT-III : The general form of Cauchy's Theorem: Chains and cycles -- Simple Connectivity - Homology - The General statement of Cauchy's Theorem - Proof of Cauchy's theorem - Locally exact differentials- Multilply connected regions - Residue theorem - The argument principle.

Chapter 4 : Section 4 : 4.1 to 4.7

Chapter 4 : Section 5: 5.1 and 5.2

UNIT-IV :Evaluation of Definite Integrals – Harmonic Functions: Definition and basic properties - Mean value property - Poisson formula.

Chapter 4 : Section 5 : 5.3

Chapter 4 : Sections 6 : 6.1 to 6.3

UNIT-V: Schwarz theorem - The reflection principle. Series and product developments: Weierstrass theorem – Taylor's Series – Laurent series.

Chapter 4 : Sections 6.4 and 6.5

Chapter 5 : Sections 1.1 to 1.3

Recommended Text :

Lars V. Ahlfors, Complex Analysis, (3rd Edition), McGraw Hill Co., New York, 1979.

Reference Books:

- 1. H. A. Presfly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.
- 2. *J.B. Conway*, Functions of one complex variables, Springer Verlag, International Student Edition, Naroser Publishing Co. 1978.
- 3. E. Hille, Analytic Function Theory (2 Vols.), Gonm & Co, 1959.
- 4. M. Heins, Complex Function Theory, Academic Press, New York, 1968.

- > Demonstrate the knowledge of functions of a complex variable.
- > Analyze the behavior of an analytic function and harmonic function in specified domain

| Title of the Course | | Topology | |
|---------------------|----------------|--------------|-------|
| Class | M Sc Maths | Subject Code | 21314 |
| Year | II | Semester | Ι |
| Course | Core Paper XII | Credit | 4 |

UNIT-I: Metric Spaces: Convergence, completeness and Baire's Theorem; Continuous mappings; Spaces of continuous functions; Euclidean and Unitary spaces. Chapter Two (Sec 12 - 15)

UNIT-II: Topological Spaces: Definition and some Examples; Elementary concepts. Open bases and subbases; Weak topologies; the function algebras C(X,R) and C(X,C): Chapter Three (Sec 16 - 20)

UNIT-III: Compact spaces, Tychonoff's theorem and locally compact spaces; Compactness for metric spaces; Ascoli's theorem.

Chapter Four (Sec 21,23,24,25)

UNIT-IV :Separation: T1 – spaces and Hausdorff spaces; Completely regular spaces and normal spaces; Urysohn's lemma and the Tietze extension theorem; The Urysohn imbedding theorem. Chapter Five (Sec 26 - 29)

UNIT-V: Connectedness: Connected spaces; The components of a space; Totally disconnected spaces; Locally connected spaces; The Weierstrass approximation Theorem. Chapter Six (Sec 31 - 34) Chapter Seven (Sec 35 only)

Recommended Text Book:

George F.Simmons, Introduction to Topology and Modern Analysis, Tata-McGraw Hill. New Delhi, 2004.

Reference Books:

- 1. James R. Munkres, Topology (2nd Edition) Pearson Education Pve. Ltd., Delhi-2002 (Third Indian Reprint)
- 2. J. Dugundji, Topology, Prentice Hall of India, New Delhi, 1975.
- 3. J.L. Kelly, General Topology, Van Nostrand, Reinhold Co., New York
- 4. S. Willard, General Topology, Addison Wesley, Mass., 1970

- > Apply the concept of convergence, completeness and continuous mappings in metric spaces.
- \blacktriangleright To demonstrate the applications of topological spaces, compact spaces and connected spaces.

| Title of the Course | | Mechanics | |
|---------------------|-----------------|--------------|-------|
| Class | M Sc Maths | Subject Code | 21315 |
| Year | II | Semester | III |
| Course | Core Paper XIII | Credit | 4 |

UNIT-I : Mechanical Systems : The Mechanical system- Generalised coordinates – Constraints - Virtual work – Energy and Momentum

UNIT-II: Lagrange's Equations: Derivation of Lagrange's equations - Examples- Integrals of motion.

UNIT-III: Hamilton's Equations: Hamilton's Principle- Hamilton's Equation - Other variational principle.

UNIT–IV: Hamilton-Jacobi Theory: Hamilton Principle function – Hamilton-Jacobi Equation -Separability

UNIT-V: Canonical Transformation: Differential forms and generating functions–Special Transformations–Lagrange and Poisson brackets.

Recommended Text Book:

D. Greenwood, Classical Dynamics, Prentice Hall of India, New Delhi, 1985.

| Chapter 1 | Sections 1.1 to 1.5 |
|-----------|--|
| Chapter 2 | Sections 2.1 to 2.3 (Omit Section 2.4) |
| Chapter 3 | Sections 4.1 to 4.3 (Omit Section 4.4) |
| Chapter 4 | Sections 5.1 to 5.3 |
| Chapter 5 | Section 5.1 to 5.10 (Omit Section 5.11), (Omit examples 5.5.1 and 5.5.2) |
| Chapter 6 | Sections 6.1, 6.2 and 6.3 (Omit Sections 6.4, 6.5 and 6.6) |

Reference Books:

- 1. H. Goldstein, Classical Mechanics, (2ndEdition) Narosa Publishing House, New Delhi.
- 2. N.C.Rane and P.S.C.Joag, Classical Mechanics, Tata McGrawHill, 1991.
- 3. *J.L.Synge and B.A.Griffth*, Principles of Mechanics(3rd Edition) McGraw Hill Book Co., NewYork, 1970.
- 4. E.T. Whittaker-A Treatise on Analytical dynamics of particles and rigid bodies.

- Use basic concepts and principles of classical mechanics, and apply them to simple examples.
- Apply the equations of motion for complicated mechanical systems using the Lagrangian and Hamiltonian formulation of classical mechanics

| Title of the Course | | Differential Geometry | |
|---------------------|----------------|-----------------------|-------|
| Class | M Sc Maths | Subject Code | 21316 |
| Year | II | Semester | III |
| Course | Core Paper XIV | Credit | 4 |

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UNIT–I: Definition of a space curve – Arc length – tangent – normal and binormal – curvature and torsion.

UNIT–II: Contact between curves and surfaces- tangent surface- involutes and evolutes. Intrinsic equations – Fundamental Existence Theorem for space curves- Helices.

UNIT-III: Definition of a surface – curves on a surface – Surface of revolution – Helicoids -

Metric - Direction coefficients – families of curves.

UNIT-IV: Geodesics – Canonical geodesic equations – Normal property of geodesics - Existence Theorems- Geodesic curvature – Gauss-Bonnet Theorem.

UNIT – V: Gaussian curvature – surfaces of constant curvature – The second fundamental form- Principal curvatures – Lines of curvature

Recommended Text Book:

T.J.Willmore, An Introduction to Differential Geometry, Oxford University Press, (17th Impression) New Delhi 2002. (Indian Print)

| Chapter 1 | Sections 1 to 9. |
|-----------|---------------------------|
| Chapter 2 | Sections 1 to 7, 10 to 18 |
| Chapter 3 | Sections 1 to 3. |

Reference Books :

- 1. Struik, D.T. Lectures on Classical Differential Geometry, Addison-Wesley, Mass.1950.
- 2. Kobayashi. S. and Nomizu. K. Foundations of Differential Geometry, Interscience Publishers, 1963.
- 3. *Wilhelm Klingenberg*, A course in Differential Geometry, Graduate Texts inMathematics, Springer-Verlag 1978.
- 4. J.A. Thorpe, Elementary topics in Differential Geometry, Under- graduate Texts in Mathematics, Springer Verlag 1979.

- Examine and describe the concepts of curves, surfaces, involutes, evolutes, surface of revolution
- > Demonstrate the application of geodesic on a surface and Gaussian curvature.

| Title of the Course | | Number Theory and Cryptography | |
|---------------------|-----------------------|--------------------------------|-------|
| Class | M Sc Maths | Subject Code | 21317 |
| Year | II | Semester | III |
| Course | Elective III-Paper XV | Credit | 5 |

Unit I: Some Topics in Elementary Number Theory: Time estimates for doing arithmetic – Divisibility and the Euclidean algorithm – Congruences – Some applications to factoring.

Unit II: Cryptography : Some simple cryptosystems – Enciphering matrices.

Unit III: Finite Fields and Quadratic Residues: Finite fields – Quadratic residues and reciprocity.

Unit IV: Public Key: The idea of Public key cryptography – RSA – Discrete log.

Unit V: Primality and Factoring: Pseudo primes - rho method. – Fermat Factorization and factor bases. The quadratic sieve method. Elliptic Curves: Basic facts – Elliptic curve cryptosystem.

Recommended Text Book:

F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers, New Delhi, 1985.

| Chapter 1 | Sections 1.1 to 1.4 |
|-----------|--------------------------------------|
| Chapter 2 | Sections 2.1 to2.2 |
| Chapter 3 | Sections 3.1 to3.2 |
| Chapter 4 | Sections 4.1 to4.3 |
| Chapter 5 | Section 5.1 to5.5 (Omit Section 5.4) |
| Chapter 6 | Section 6.1 to6.2 |

Reference Books:

- 1. *I.Niven and H.S. Zuckermahn*, An Introduction to Theory of Numbers (Edn.3), Wiley Eastern Ltd., New Delhi 1976.
- 2. David M.Burton, Elementary Number Theory, Brown Publishers, Dubuque, Iowa, 1989.
- 3. *K.Ireland and. Rosesn*, A classical Introduction to Modern Number Theory, springer verlog, 1972.
- 4. *N.Koblitz*, Algebraic aspect of (cryptography), Springer 1998.

- Restate the number theory concept with reference to numbers in different bases and categorize the Euclidean Algorithm and congruences by analyzing their characteristics and structure.
- > Demonstrate the application of some simple cryptosystem using matrices.
- > Prepare a formula for primality test and application of elliptic curves in cryptography.

| Title of the Course | | Complex Analysis II | |
|---------------------|----------------|---------------------|-------|
| Class | M Sc Maths | Subject Code | 21419 |
| Year | II | Semester | IV |
| Course | Core Paper XVI | Credit | 4 |

UNIT-I : Partial fractions & Factorization: Partial fractions – Infinite products – canonical products – The Gamma functions – Jensons formula.

Chapter 5 : Sections 2.1 to 2.4 (omit 2.5)

Chapter 5 : Sections 3.1 (omit 3.2)

Unit- II: The Riemann Zeta functions: The product development – Extension of (s) to the whole plane – The functional equation – Normal families :Equicontinuity – Normality and compactness – Arzela's theorem – Families of analytic functions.

Chapter 5 : Sections 4.1 to 4.3 (omit 4.4) Chapter 5 : Sections 5.1 to 5.4 (omit 5.5)

UNIT-III : The Riemann mapping Theorem : Statement and Proof – Boundary Behaviour – Use of the Reflection Principle - TheBehaviour at an angle – The Schwarz-Christoffel formula – A closer look at the Hormonic functions : Functions with mean value property –Harnack's principle. Chapter 6 : Sections 1.1 to 1.3 (Omit Section1.4) Chapter 6 : Sections 2.1 to 2.2 (Omit section 2.3 & 2.4)

Chapter 6 : Section 3.1 and 3.2.

UNIT-IV : Elliptic functions: Representation by exponentials – The Fourier development – Functions of finite order – The Period module – Unimodular transformations - The canonical basis – General properties of elliptic functions.

Chapter 7 : Sections 1.1 to 1.3 Chapter 7 : Section 2.1 to 2.4

UNIT-V: The Weierstrass Theory :The Weierstrass-function – The functions(z) and (z) – The differential equations– The modular functions.

Chapter 7 : Sections 3.1 to 3.4(Omit 3.5)

Recommended Text Book:

Lars V. Ahlfors, Complex Analysis, (3rd edition) McGraw Hill Co., NewYork, 1979

Reference Books

1.H.A. Presfly, Introduction to complex Analysis, Clarendon Press, oxford, 1990.

2.J.B. Conway, Functions of one complex variables, Springer Verlag International student Edition, Naroser PublishingCo.1978

3.E. Hille, Analytic function Thorey (2 vols.), Gonm & Co,1959.

4.M.Heins, Complex function Theory, Academic Press, NewYork, 1968

Course Outcomes: At the end of the course, the student will be able to

Determine the infinite product representation of Riemann zeta function and explore Riemann mapping theorem for simply connected regions. Understand and apply the properties of an elliptic function and in particular the Weierstrass function.

| Title of the Course | | Functional Analysis | |
|---------------------|-----------------|---------------------|-------|
| Class | M Sc Maths | Subject Code | 21420 |
| Year | II | Semester | IV |
| Course | Core Paper XVII | Credit | 4 |

UNIT-I :Banach Spaces :Definition – Some examples –Continuous Linear Transformations – The Hahn-BanachTheorem.

Chapter 9 : Sections 46-48

UNIT-II :Banach Spaces:The natural embedding of N in N** -Open mapping theorem – conjugate of an operator.

Chapter 9 : Sections 49-51

UNIT-III :Hilbert Spaces:Definition and some simple properties – Orthogonal complements – Orthonormal sets Conjugate space H*

Chapter 10 : Sections 52-55.

UNIT-IV : Hilbert Spaces: Adjoint of an operator – Self-adjoint operator – Normal and Unitary Operators – Projections

Chapter 10 : Sections 56-59.

UNIT-V: BanachAlgebras :Definition and some examples – Regular and singular elements – Topological divisors of zero – spectrum – the formula for the spectral radius – the radical and semi-simplicity.

Chapter 12: Sections 64-69

Recommended Text Book:

George F.Simmons, Introduction to Topology and Modern Analysis, Tata-McGraw Hill. New Delhi, 2004.

Reference Books:

- 1. W. Rudin, Functional Analysis, Tata McGraw-Hill Publishing Company, New Delhi, 1973
- 2. G. Bachman & L.Narici, Functional Analysis, Academic Press, New York ,1966.
- 3. *H.C. Goffman and G.Fedrick*, First course in Functional Analysis, Prentice Hall of India, New Delhi, 1987
- 4. *E. Kreyszig*, Introductory Functional Analysis with Applications, John Wiley& Sons, New York., 1978.

- > Apply the concepts of Banach spaces, Hilbert spaces and Banach Algebras.
- To explain the importance of the Hahn-Banach theorem, Open mapping theorem in Functional Analysis.

| Title of the Course | | Tensor Analysis and Theory of Relativity | |
|---------------------|------------------|--|-------|
| Class | M Sc Maths | Subject Code | 21421 |
| Year | II | Semester | IV |
| Course | Core Paper XVIII | Credit | 4 |

UNIT-I: Physical Laws – spaces of N dimensions – coordinate transformations – the summation convention – contravariant and covariant vectors – contravariant, covariant and mixed tensors – the kronecker delta – tensors of rank greater than two – scalars or invariants – tensor fields – symmetric and skew-symmetric tensors – fundamental operations with tensors.

Unit-II: Matrices – Matrix algebra – the line element and metric tensor – conjugate or reciprocal tensors – associated tensors – length of a vector – angle between vectors – physical components – Christoffel symbols.

Unit-III: Transformation laws of christoffel's symbols – Geodesics

– covariant derivatives – permutation symbols and tensors – Tensor form of gradient, divergence and curl – the intrinsic or absolute derivative – Relative and absolute tensors.

UNIT-IV: Introduction - Galilean Transformations - Maxwell's equations - The ether Theory - The Principle of Relativity – Relativistic Kinematics: Lorentz Transformation equations-Events and simultaneity - Example – Einstein Train – Time dilation – Longitudinal Contraction–The Invariant Interval–Proper time and Proper distance – The World line – Example – The twin paradox– addition of velocities – The Relativistic Doppler effect– Examples.

UNIT-V: Relativistic Dynamics, Momentum – Energy – The Momentum-energy four vector – Force – Conservation of Energy – Mass and energy – Example – inelastic collision – The Principle of equivalence – Lagrangian and Hamiltonian formulations - Accelerated Systems: Rocket with constant acceleration – example – Rocket with constant thrust

Recommended Text Book:

- 1. *Murry R. Spiegel*, Theory and problems of vector and Tensor analysis, Mcgraw Hill Book Company (Units I, II, III Chapter 8)
- 2. Donald T.Greenwood, Classical Dynamics Prentice Hall of India Pvt Ltd1990. (Units IV, V Chapter 7.1, 7.2, 7.3, 7.4)

- Formulate and express a physical law in terms of tensors and simplify it by use of the common form which is independent of the reference coordinate system
- > Restate the Relativistic Doppler effect with examples.

| Title of the Course | | Fluid Dynamics | |
|---------------------|-----------------------|----------------|-------|
| Class | M Sc Maths | Subject Code | 21422 |
| Year | II | Semester | IV |
| Course | Elective IV-Paper XIX | Credit | 5 |

Unit I: Kinematics of fluids in motion:-

Real fluids and Ideal fluids – velocity of a fluid at a point – Streamlines and Path lines - Steady and Unsteady flows - Velocity Potential – Vorticity vector - Local and Particle rates of change - Equation of Continuity – Examples: Acceleration of a fluid – Conditions at a Rigid Boundary.

Unit II Equations of motion of a fluid: Pressure at a point in a fluid at rest – pressure at a point in a moving fluid – Conditions at a Boundary of two invisid immiscible fluids – Enler's equation of motion – Bernoulli's equation. Example: Discussion of the case of study motion under Conservative body focus.

Unit III Some three – Dimensional flows: Introduction – Sources, Sinks and doublets – Images in a Rigid Infinite plane – Axi – symmetric flows – Stoke's stream function.

Unit IV Some Two – Dimensional flows: Meaning of two – dimensional flow – Use of Cylindrical Polar Coordinates – Stream function – Complex Potential for Two – Dimensional, Irrotational, Incompressible flow – complex velocity potentials for standard two – Dimensional flows – Uniform stream, Line Sources and Line Sinks, Line Doublets, Line vortices – Examples – Two – Dimensional Image Systems – Milne Thomson Circle theorem – Applications of Circle therein, Extension of Circle therein.

Unit V Viscous flow: Stress Components in a real fluid – Relation between Cartesian components of stress – Translational motion of fluid clement – The Rate of strain Quadric and principal stresses – Some further properties of Rate of strain Quadric - stress Analysis in fluid motion – Relations between Stress and Rate of strain – Coefficient of viscosity and Laminar flow – Navier – Stokes equation of motion of viscous fluid.

Recommended Text Book:

F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers, New Delhi, 1985.

| Chapter 2 | Sections 2.1 to 2.10 |
|-----------|--|
| Chapter 3 | Sections 3.1 to 3.7 |
| Chapter 4 | Sections 4.1 to 4.5 (omit Section 4.4) |
| Chapter 5 | Section 5.1 to 5.8 |
| Chapter 8 | Section 8.1 to 8.9 |

- Restate the kinematics of a moving fluid.
- > Demonstrate the applications of Milne Thomson Circle theorem with examples.

| Title of the Course | | Integral Equations and Calculus of variations | |
|---------------------|----------------------|---|-------|
| Class | M Sc Maths | Subject Code | 21423 |
| Year | II | Semester | IV |
| Course | Elective IV-Paper XX | Credit | 5 |

UNIT-I: Integral Equations: Introduction – Integral equation Definition – Linear and nonlinear integral equations – Fredholm integral equation – Volterra Integral equations – Special kinds of kernels – Resolvent kernel or reciprocal kernel – Eigen values and Eigen vectors – Solution of an integral equation – Solved examples based on integral equations

Initial value problem – Method of converting an initial value problem into a Volterra integral equation – Alternative method of converting an initial value problem in to a Volterra integral equation.

Unit-II: Boundary value problem – Method of converting a boundary value problem into a Fredholm integral equation – Characteristic Values – Characteristic functions – Solution of homogeneous Fredholm integral equation of the second kind with separable kernels – Solved examples.

Unit-III: Solution of Fredholm integral equations of the second kind with separable (or degenerate) kernels – Solved examples.

Unit-IV: Calculus of Variations: The concept of Variation and its properties – Euler's equation – Variational problems for specific functional – Functionals dependent on Higher-Order derivatives – Variational problems in parametric form – Some applications to problem of Mechanics.

UNIT-V: Variational problems with moving Boundaries: Functionals of special type – Variational problems with moving boundary for a functional dependent on two functions.

Sufficient conditions for an Extremum: Field of extremals – Jacobi conditions – Weirstrass function – Legendre condition – Second Variation.

Recommended Text Book:

- **1.** *M.D.Raisinghania*, Integral Equations and Boundary Value Problems by, S.Chand Publishers, First Edition 2007(For Units I, II and III)
- **2.** *A.S.Gupta*, Calculus of Variations with Applications, Prentice Hall of India Private Limited, New Delhi -1, Seventeenth Printing April 2008. (For Units IV and V)

| | Integral Equations |
|-----------|--|
| Chapter 1 | Sections 1.1 to 1.18 (Omit sections 1.2, 1.7, 1.9, 1.10, 1.13, 1.14, 1.15, 1.16) |
| Chapter 2 | Sections 2.2 to 2.4, 2.5, 2.6 (Omit section 2.1) |
| Chapter 3 | Sections 3.1 to 3.3 |
| Chapter 4 | Sections 4.1, 4.2 |
| | Calculus of variations |
| Chapter 1 | Sections 1.1 to 1.7 (Omit section 1.5) |
| Chapter 2 | Sections 2.1 and 2.2 Chapter 3 : Sections 3.1 to 3.5 |
| Chapter 3 | Chapter 3 : Sections 3.1 to 3.5 |

Reference Books:

1. M.K. Venkatraman, Higher Engineering Mathematics.

2. L. Elsgolts, Differential equations and the calculus of variations, Mir Publishers, Moscow, 1970.

- > Recognize and solve initial value problems and boundary value problems
- > Explore the methods for finding extreme values of functionals

Demonstrate the application of variational problems with moving boundaries and field of extremals using Jacobi and Legendre conditions

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